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A ROTATING OSCILLATING STREAMING FLUID JET'S STABILITY UNDER SELF-GRAVITATING FORCE

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ABSTRACT: *We address the stability of a rotating fluid jet that oscillates and is self- gravitating. The fundamental equations are resolved, and the problem is defined. An analytically developed and numerically verified general Eigen-value relation is derived. While the fluid jet is unstable for small wave numbers in the ax symmetric mode, it is entirely stable in the non-ax symmetric perturbation. In both the ax symmetric and non-ax symmetric modes, the stable states are reduced as a result of streaming. For a narrow range of wave numbers, the self- gravitating force is only destabilizing in the symmetric mode (m=0), but it stabilizes all other perturbations. Academically speaking and during the geological drilling in the*

KEYWORDS: *Capillary force; self gravitating; Hydro magneticIntroduction*

Chandrasekhar provides the first classical description of the capillary instability of a gas cylinder submerged in a liquid under an ax symmetric perturbation. Kruskal & Tuck (I 938), among others, recently looked into the stability of a cylindrical plasma (the pinch') with an axial magnetic field. In particular, Rosenbluth has demonstrated how the existence of an axial magnetic field can, under the right conditions, maintain the pinch when the plasma is contained between conducting walls. Additionally, Rosenbluth has approached the issue from the viewpoints of both conventional hydromagnetics (with the usual assumptions of scalar pressure and adiabatic changes of state) and the physically more significant viewpoint of the orbits described by the ions and electrons in the external magnetic field. Chandrasekhar hasproven the MHD stability of a full fluid cylinder that is surrounded by a uniform magnetic field. Kendall carried out experiments to gather and analyze annular fluid jet stability.

Additionally, he attracted and drew interest in examining the general stability of this model due to its critical astrophysical implications. Chandrasekhar provides the first classical description of the capillary instability of a gas cylinder submerged in a liquid under an ax symmetric perturbation. The dispersion relation was provided by Hasan, Elazab et al., Drazin and Reid, and it is applicable to all ax-symmetric and non-ax- symmetric modes. In all kinds of perturbation, Cheng examined the instability of a gas jetin an incompressible liquid. However, it's important to note that The Sirignano talk about narrow annular liquid sheets with axisymmetric capillary waves. Self-gravitating stability of a fluid cylinder contained in a confined liquid, permeated by a magnetic field, for all symmetric and asymmetric perturbation modes is the goal of the current work.

FORMULATION OF THE PROBLEM

We take a look at an infinite circular Fluid jet in a spinning, oscillating cylinder with an oscillating velocity $\underline{U} = (0, 0, U\, e^{\omega t})$ and rotating with angular velocity $\underline{\Omega} = (0,\ 0,\ \Omega)$ thefluid is assumed to be non-viscous, incompressible and perfectly conducting. We shall use a cylindrical polar coordinates $(r,\ \varphi,\ z)$ system with the z-axis coinciding with the axis of the annular jet. The basic equation are the hydro magnetic equation of motion,

continuity equation.

The equation of motion (1)

Continuity equation

$$\rho \left(\frac{\partial u}{\partial t} \nabla (\underline{u}\nabla)\underline{u}\right) = -\nabla P + \nabla V^i + \frac{1}{2}\nabla\left|\underline{\Omega} \wedge \underline{r}\right|^2 + 2(\underline{u} \wedge \underline{\Omega}) \quad (2)$$

The Poisson's equation satisfying the gravitational field interior the fluid, and Laplace'sequation satisfying the gravitational potential of the medium surrounding the fluid cylinder. $\nabla^2 V^i = -4\pi G \rho$ (3)

$\nabla^2 V^e = 0$ (4)

Where $(\underline{U})$ and (ω) are the amplitude and oscillation frequency of the velocity, ρ, P are the mass density and kinetic pressure; G is the gravitational constant, V^i is the gravitational potential interior the fluid cylinder and V^e the gravitational potential exteriorthe fluid cylinder.

EQUILIBRIUM STATE

In the unperturbed state the system of the basic equation (1) – (4) take the form

$$\nabla\left(\frac{p_0}{\rho} - \frac{1}{2}\nabla\left|\underline{\Omega} \wedge \underline{r}\right|^2 - V_0^i\right) \quad (5)$$

$$\underline{u_0} = 0, \qquad \nabla \cdot \underline{u_0} = 0 \quad (6)$$

$$\nabla^2 V_0^i = -4\pi G \rho \quad (7)$$

$$\nabla^2 V_0^e = 0 \quad (8)$$

These equation are simplified with $\frac{\partial}{\partial z} = 0$ and $\frac{\partial}{\partial \varphi} = 0$ and the resulting system aftersimplification is solved. The solution obtained are matched at $r = R_0$ across the boundary surface of the fluid. The finite solution can be easily found.

PERTURBATION ANALYSIS

For small departure from the unperturbed state, every physical quantity $Q(r, \varphi, z, t)$ couldbe expressed as .

$$Q(r, \varphi, z; t) = Q_0(r) + \varepsilon_0(t) Q_1(r, \varphi, z) \quad (9)$$

Where Q_1 stands for P, $\underline{U}$, V^i, V^e, the amplitude of perturbation $\varepsilon(t)$ at time t is

$$\varepsilon(t) = \varepsilon_0 \exp(\sigma t) \quad (10)$$

Where (σ)is the growth rate of the instability or rather the oscillation frequency if ($\sigma = iw \; with \; i = \sqrt{-1}$) is imaginary and ε_0 is the amplitude at $t = 0$ The perturbed radii distances f the gas cylinder is given by where

$$r = R_0 + \varepsilon_0 \, exp(\sigma t + i(kz + m\phi)) \quad (11)$$

Where (k) is the longitudinal wave number and (m an integer) is the transverse wave number. The second term on the right-hand side of equation (11) represent the surface wave elevation normalized with respect to R_0 and measured from the equilibrium position.

The linearized perturbation equation deduced from the fundamental equations (1)- (4)are given by

$$\frac{\partial \underline{U_1}}{\partial t} + \left(\underline{U} . \underline{\nabla}\right)\underline{U_1} = -\underline{\nabla}\Gamma + 2\left(\underline{U_1} \wedge \underline{\Omega}\right) \quad (12)$$

$$\Gamma = \frac{P_1}{\rho} - V_1^i \quad (13)$$

$$\underline{\nabla} . \underline{U_1} = 0 \quad (14)$$

$$\nabla^2 V_1^i = 0 \quad (15)$$

$$\nabla^2 V_1^e = 0 \quad (16)$$

This system of equation is simplified on using the time dependence as given above by (10). From the view point of the linear theory and based on the linear perturbation technique, every perturbed quantity can be expressed as $exp(\sigma t + i(kz + m\phi))$ times an amplitude function of r. Consequently, on solving (12) we obtain

$$u_{1r} = \frac{-(\sigma+ikUe^{\omega t})}{\rho[(\sigma+ikUe^{\omega t})^2+4\Omega^2]} \cdot \frac{\partial \Gamma}{\partial r} + \frac{4im\Omega^2}{\rho[(\sigma+ikUe^{\omega t})^2+4\Omega^2]} \cdot \frac{\Gamma}{r} \tag{17}$$

$$u_{1\phi} = \frac{2}{\rho[(\sigma+ikUe^{\omega t})^2+4\Omega^2]} \cdot \frac{\partial \Gamma}{\partial \phi} + \frac{im\Gamma}{r(\sigma+ikUe^{\omega t})} \cdot \left[\frac{4\Omega^2}{\rho[(\sigma+ikUe^{\omega t})^2+4\Omega^2]} - 1 \right] \tag{18}$$

$$u_{1z} = \frac{-ik\Gamma}{(\sigma+ikUe^{\omega t})} \tag{19}$$

From equation (14) and (17,18,19) we get

$$\frac{d^2\Gamma}{dr^2} + \frac{1}{r}\frac{d\Gamma}{dr} + \left(q - \frac{m^2}{r^2}\right) = 0 \tag{20}$$

$$q = k^2\left[1 + \frac{4\Omega^2}{(\sigma+ikUe^{\omega t})^2}\right] \tag{21}$$

The solution of equation (20) is given by

$$\Gamma = A\, J_m(qr) exp(i(kz + m\phi + \sigma\, t)) \quad (22)$$

Where A is an arbitrary constant to be determined and J_m the ordinary Bessel function offirst kind of order m. Similarly equation (15) and (16), based on the linearzed theory, are solved and first orderperturbation V_1^i and V_1^e are given by

$$V_1^i = B\, I_m(kr) exp(i(kz + m\phi + \sigma\, t)) \tag{23}$$

$$V_1^e = C\, K_m(kr) exp(i(kz + m\phi + \sigma\, t)) \tag{24}$$

Where I_m and K_m are modified Bessel function of order m and B, C are constants of integrations to be determined.

BOUNDARY CONDITION

The Solution represented by equations (22) – (24) must satisfy certain boundary conditions. Under the present circumstances these conditions can be given as follows.

(i) Kinematics boundary condition states that " The normal component of the velocity U_{1r} vector must be compatible with the velocity of the particles ofthe boundary surface at $r = R_0$

$$U_{1r} = \frac{\partial r}{\partial t} = \frac{\partial \Gamma}{\partial r} \qquad (25)$$

(ii) The gravitational potential and its derivatives must be continuous across thesurface.

(iii) The normal component of the total stress tensor must be continuous across theboundary surface from which we have the following dispersion relation:

$$\frac{(\sigma+ikUe^{\omega t})[(\sigma+ikUe^{\omega t})+4\Omega^2]}{[yJ_m(y)(\sigma+ikUe^{\omega t})+2\imath mJ_m(y)]} J_m(y) = 2\pi G\rho - \Omega^2 - 4\pi G\rho K_m(x) I_m(x) \qquad (25)$$

Where is, the dimensional wavenumber, given by $x = kR_0$ and is defiend by $y = qR_0$.
Equation (25) is the dispersion relation of gravitational streaming oscillating rotating fluid cylinder surrounded by self-gravitating vacuum. It relates the growth rate N with thestreaming oscillating velocity $Ue^{\omega t}$, angular velocity Ω, the wave numbers x, y, m and other parameters ρ, G and R_0.

STABILITY DISCUSSION

It is advisable to analyze the behaviors of the Bessel functions as well as those of the compound functions contained in the relation before we discuss the ordinary stability, marginal stability, and instability of the system under examination (25). Considering the
)recurrence relationships (see Abramowitz and Stegun

$$2\, I'_m(x) = I_{m-1}(x) + I_{m+1}(x), \qquad (26)$$

$$2\, K'_m(x) = -K_{m-1}(x) - K_{m+1}(x) \qquad (27)$$

Because $I_m(x)$ is monotonic increasing and positive definite $I_m(x) > 0$ for all modes of perturbation $m \geq 0$ and nonzero values of $x \neq 0$ while $K_m(x)$ is monotonically decreasing but never negative, i.e., $K_m(x) > 0$ we may show that

$$I'_m(x) > 0, \qquad K'_m(x) < 0 \qquad (28)$$

Also for $m \geq 1$ for all values of $x \neq 0$, *we have*

$$2I_m(x)K_m(x) < 1 \qquad (29)$$

For non-rotating $(i.e\ \Omega = 0)$ and non-streaming fluid $(i.e\ U = 0)$ the dispersion relation (25) reduces to that of Chandraskhar . Moreover if we put $m = 0$, $\Omega = 0$, $U = 0$ in (25) werecover the relation derived their relation by using the principle of Fermi .
In absence of the streaming $(i.e\ U = 0)$, the dispersion relation (25) can be written in thedimensionless form

$$[yNJ'_m(y) + 2mMJ_m(y)]\left[K_m(x)I_m(x) - \frac{1}{2} + M^2\right] + N(N^2 + 4M^2)J_m(y) = 0$$

(30)

Where the dimensionless quantity M, N we defined as followIf the dispersion relation (30) takes the simpler form

$$N = \frac{1}{\sqrt{4\pi G\rho}}, \qquad M = \frac{\Omega}{\sqrt{4\pi G\rho}}. \qquad (31)$$

$$x = 0 \qquad (32)$$

Hence we get $N^2 - 2MN + m\left(M^2 - \frac{1}{2}\right) + \frac{1}{2} = 0$. (33)

A neutral mode of oscillation is obtained if

$$M = \pm\frac{\sqrt{(m-1)}}{\sqrt{2m}}, \qquad N = M \pm \sqrt{(m-1)(\frac{1}{2} - M^2)}. \qquad (34)$$

It is clear that the angular velocity must satisfy the following $$\sqrt{2}\,\mathbf{M} \leq \mathbf{1} \qquad (35)$$
Neglecting the rotation effect the dispersion relation (25) reduces to
$$(\sigma + ikUe^{\omega t})^2 = \frac{4\pi G\rho x I'_m(x)}{I_m(x)}\left[I_m(x)K_m(x) - \frac{1}{2}\right]. \qquad (36)$$

CONCLUSION

1- The streaming has the effect of lowering the stable states in both ax symmetric andnon-ax symmetric modes..
2- The self-gravitating force stabilizes for all other perturbations but only destabilizes inthe symmetric mode (m=0) for a narrow range of wave numbers.
3- When the destabilizing behavior of the model is diminished and inhibited, the stabilitybehavior of the model follows.
4- Because we have superimposed gas-oil layer mixed fluids, this phenomena isintriguing to researchers and is observed during geological drilling in the earth's crust.

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