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Mathematical modeling of a Production planning and Product storage problem

Mai Nguyen Thi Xuan
Faculty of Fundamental and Applied Sciences, Thai Nguyen University of Technology, Vietnam

ABSTRACT: *In recent years, mathematics has been increasingly applied in a wide range of fields, demonstrating its significant influence on economics, engineering, transportation, information technology, and many other sciences. In order to clarify the relationship between mathematics and practical problems, this paper discusses a real-world problem whose solution requires mathematical modeling. Specifically, the problem concerns the formulation of a production and product storage plan that minimizes the total incurred cost.*

Keywords: *real-world problem, optimal solution, modeling, linear programming problem, constraints, minimization*

INTRODUCTION

At present, scientific research oriented toward practical applications and the solution of problems arising in practice has attracted considerable attention. One of the most important directions in this regard is optimization theory. In actual production, there are many types of products for which social demand varies over time, while the storage of such products requires relatively high costs. For example, in cement production, the demand for cement during the dry season for construction purposes is very high; however, storage costs during the rainy season, when the climate is humid, must also be taken into account. Therefore, if production and design planning are based on optimality principles, it is possible to reduce costs related to capital, raw materials, time, and labor, while improving efficiency, productivity, and the quality of work.

The problem is to model practical situations as optimization problems, thereby determining a production process for which the total cost arising from production fluctuations and product storage is minimized. The solution of the optimization problem provides the most reasonable and advantageous production and storage plan in practice.

THE PRODUCTION PLANNING AND PRODUCT STORAGE PROBLEM

The production planning and product storage problem in an enterprise is of great importance because it determines the operation of the entire manufacturing plant. As a result, it helps improve production efficiency, increase profits, and enhance workers' income.

To formulate a production plan, the production department must coordinate closely with the marketing department, the human resources department, and the warehouse management department in order to collect and analyze relevant information. Some important information includes:

- The production capacity of the plant, including the capacity of machines and equipment, as well as maintenance and repair schedules.
- Material consumption norms and the capacity of each production stage.
- The current inventory levels of goods and materials.
- Information on personnel and workers who can participate in production at the plant.
- The number of orders already placed and the required delivery times.

Based on the above information, the plant will develop a specific plan according to criteria such as:

- The quantity of additional materials that need to be ordered.
- Whether overtime work is required to meet the production schedule, and if so, whether the profit is sufficient to cover the additional labor cost.
- Whether surplus production should be carried out for storage purposes.
- The costs of storage and warehousing for finished products.

The above criteria are optimized within a problem called the production planning and product storage problem. This is a multi-period optimization problem that balances production costs and inventory costs with market demand over time. In practice, if production is carried out exactly according to monthly requirements or individual orders, production costs may increase considerably in months with high demand, while production equipment may be underutilized in months with low demand. Conversely, if production is maintained at a stable level so that surplus products in some months compensate for shortages in other months, additional costs for product storage will be incurred. Therefore, it is necessary to determine a production plan that minimizes the total cost arising from production fluctuations and storage.

MATHEMATICAL MODELING OF THE PRODUCTION PLANNING AND PRODUCT STORAGE PROBLEM

Let:

I_t denote the inventory level in month t. Thus, I_0 denotes the inventory level before production begins.

X_t denote the quantity of products produced in month t.

R_t denote the market demand in month t.

S_t denote the quantity of products stored after month t.

It is easy to see that the quantity of products stored after month t is equal to the total quantity produced plus the previous inventory, minus the demand in month t. Hence, we have the following relationship:

$X_t + I_{t-1} - R_t = I_t$ or equivalently $X_t + I_{t-1} - I_t = R_t$

Therefore $X_t - X_{t-1} + I_1 = R_t$.

The difference $X_t - X_{t-1}$ represents the change in production output between two consecutive months, indicating whether the company expands or reduces its production scale.

Set $X_t - X_{t-1} = Y_t$

If $Y_t \geq 0$, then Y_t represents the degree of production expansion; if $Y_t \leq 0$, then Y_t represents the degree of production reduction.

Thus, the system of constraints of the model is obtained as follows:

$$\begin{cases} X_t + I_{t-1} - I_t = R_t \\ X_t - X_{t-1} - Y_t = 0 \\ X_t \geq 0;\ I_t \geq 0\ ; t = 1,2,\ldots \end{cases}$$

Suppose that, based on production experience from previous years, the cost of expanding production in month t compared with month t - 1 is known to be a monetary units per unit of product, and the storage cost of one unit of product for one month is b monetary units. Then the total production and storage cost is

$T = a.\sum_{t=1}^{n} X_t + b.\sum_{t=1}^{n} I_t$.

Therefore, we obtain the following linear programming problem:

Minimize the total cost $T = a.\sum_{t=1}^{n} X_t + b.\sum_{t=1}^{n} I_t$ s subject to the constraints

$$\begin{cases} X_t + I_{t-1} - I_t = R_t \\ X_t - X_{t-1} - Y_t = 0 \\ X_t \geq 0;\ I_t \geq 0\ ; t = 1,2,\ldots \end{cases}$$

ILLUSTRATIVE EXAMPLE

To illustrate the above linear programming problem, we consider the following example.

In preparation for the Mid-Autumn Festival, a confectionery company needs to formulate a production and storage plan for mooncakes over three months: July, August, and September. Based on a market demand survey and production experience from previous years, the company forecasts the following demand:

- July: market demand is approximately 1,000 boxes.
- August: market demand is approximately 3,000 boxes.
- September: market demand is approximately 2,000 boxes.

Regarding costs, the estimated production cost is 100,000 VND per box, and the end-of-month storage cost is 10,000 VND per box.

Regarding production capacity, the company can produce at most 2,500 boxes per month. Assume that the initial inventory is 0 boxes, meaning that there is no remaining inventory from the previous period. The objective is to formulate a production plan that minimizes the total production cost while still satisfying consumer demand.

We now model the problem mathematically.

Let X_i denote the number of products produced in month i, where $i = 7; 8; 9$.

Let I_i denote the inventory level at the end of month i, where $i = 7, 8, 9$. Since the initial inventory is assumed to be 0 boxes, we have $I_0 = 0$.

Let R_i denote the market demand in month i, where $i = 7, 8, 9$. Thus,

$R_7 = 1000; R_8 = 3000;\ R_9 = 2000$.

The total production and storage cost is then

$T = 100000.(X_7 + X_8 + X_9) + 10000.(I_7 + I_8 + I_9)$.

The constraints of the problem are as follows:

1. Inventory balance. That is, the total quantity produced and the initial inventory must be equal to the total demand and the ending inventory. Specifically:

July: $X_7 + 0 = 1000 + I_7 \rightarrow X_7 - I_7 = 1000$.

August: $X_8 + I_7 = 3000 + I_8 \rightarrow X_8 + I_7 - I_8 = 3000$.

September: $X_9 + I_8 = 2000 + I_9 \rightarrow X_9 + I_8 - I_9 = 2000$.

2. Production capacity constraints: $X_7 \leq 2500;\ X_8 \leq 2500;\ X_9 \leq 2500$

3. Non-negativity constraints for production quantities and inventory levels: $X_i, I_i \geq 0$.

Therefore, the mathematical model of the problem is to minimize the production cost function.

$$T = 100000.(X_7 + X_8 + X_9) + 10000.(I_7 + I_8 + I_9)$$

subject to the constraints

$$\begin{cases} X_7 - I_7 = 1000 \\ X_8 + I_7 - I_8 = 3000 \\ X_9 + I_8 - I_9 = 2000 \\ X_7 \leq 2500;\ X_8 \leq 2500;\ X_9 \leq 2500 \\ X_i, I_i \geq 0\ ;\ \mathrm{i} = 7; 8; 9 \end{cases}$$

In practice, to solve this problem, the company recognizes that the cost of inventory storage is much lower than the cost associated with failing to meet market demand or being constrained by production capacity. Therefore, the usual strategy is to produce at maximum capacity in the earlier months in order to reduce high production pressure in the final month, if any, or to distribute production costs more evenly. Moreover, the conditions considered in the problem are idealized assumptions. In actual production, the company must also take into account labor costs, which may vary when working shifts increase or decrease, as well as changes in raw material prices over time.

CONCLUSION

This paper has presented a practical application of mathematics, namely the mathematical modeling of a production planning problem aimed at optimizing production and product storage costs. This further highlights the close two-way relationship between mathematics and practice, as well as the important role of mathematics in real-world applications. However, within the scope of this paper, we have only considered the modeling of the problem. The development of an optimal method for solving this problem will be examined in another paper.

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